# 6-2 Multiplying Polynomials

**TEKS 2A.2.A Foundations for functions: use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.**

***Objectives***

Multiply polynomials.

Use binomial expansion to expand binomial expressions that are raised to positive integer powers.

**Who uses this?**

Business managers can multiply polynomials when modeling total manufacturing costs. (See Example 3.)

To multiply a polynomial by a monomial, use the Distributive Property and the Properties of Exponents.

## EXAMPLE 1 Multiplying a Monomial and a Polynomial

**Remember!** To review Properties of Exponents, refer to Lesson 1-5.

**Find each product.**

**A** $3x^2(x^3 + 4)$

$3x^2(x^3 + 4)$

$3x^2 \cdot x^3 + 3x^2 \cdot 4$ *Distribute.*

$3x^5 + 12x^2$ *Multiply.*

**B** $ab(a^3 + 3ab^2 - b^3)$

$ab(a^3 + 3ab^2 - b^3)$

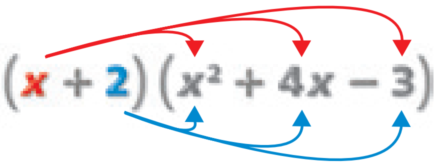
$ab(a^3) + ab(3ab^2) + ab(-b^3)$

$a^4b + 3a^2b^3 - ab^4$

**CHECK IT OUT!** **Find each product.**

**1a.** $3cd^2(4c^2d - 6cd + 14cd^2)$ **1b.** $x^2y(6y^3 + y^2 - 28y + 30)$

To multiply any two polynomials, use the Distributive Property and multiply each term in the second polynomial by each term in the first.




Keep in mind that if one polynomial has $m$ terms and the other has $n$ terms, then the product has $mn$ terms before it is simplified.

## EXAMPLE 2 Multiplying Polynomials

**Find each product.**

**A** $(x - 2)(1 + 3x - x^2)$

**Method 1** Multiply horizontally.

$(x - 2)(-x^2 + 3x + 1)$ *Write polynomials in standard form.*

$x(-x^2) + x(3x) + x(1) - 2(-x^2) - 2(3x) - 2(1)$ *Distribute x and then −2.*

$-x^3 + 3x^2 + x + 2x^2 - 6x - 2$ *Multiply. Add exponents.*

$-x^3 + 5x^2 - 5x - 2$ *Combine like terms.*

**Method 2** Multiply vertically.

| | |
|---|---|
| $-x^2 + 3x + 1$ | *Write each polynomial in standard form.* |
| $\underline{\phantom{-x^2 + 3}x - 2}$ | |
| $2x^2 - 6x - 2$ | *Multiply $(-x^2 + 3x + 1)$ by $-2$.* |
| $\underline{-x^3 + 3x^2 + x\phantom{-2}}$ | *Multiply $(-x^2 + 3x + 1)$ by $x$, and align like terms.* |
| $-x^3 + 5x^2 - 5x - 2$ | *Combine like terms.* |

**Find each product.**

**B** $(x^2 + 3x - 5)(x^2 - x + 1)$

Multiply each term of one polynomial by each term of the other. Use a table to organize the products.

| | $x^2$ | $-x$ | $+1$ |
|---|---|---|---|
| $x^2$ | $x^4$ | $-x^3$ | $+x^2$ |
| $+3x$ | $+3x^3$ | $-3x^2$ | $+3x$ |
| $-5$ | $-5x^2$ | $+5x$ | $-5$ |

The top left corner is the first term in the product. Combine terms along diagonals to get the middle terms. The bottom right corner is the last term in the product.

$x^4 + (3x^3 - x^3) + (-5x^2 - 3x^2 + x^2) + (5x + 3x) + (-5)$

$x^4 + 2x^3 - 7x^2 + 8x - 5$

**Helpful Hint**

When using a table to multiply, the polynomials must be in standard form. Use a zero for any missing terms.

CHECK IT OUT!

**Find each product.**

**2a.** $(3b - 2c)(3b^2 - bc - 2c^2)$ **2b.** $(x^2 - 4x + 1)(x^2 + 5x - 2)$

## EXAMPLE 3 *Business Application*

**Mr. Silva manages a manufacturing plant. From 1990 through 2005, the number of units produced (in thousands) can be modeled by $N(x) = 0.02x^2 + 0.2x + 3$. The average cost per unit (in dollars) can be modeled by $C(x) = -0.002x^2 - 0.1x + 2$, where $x$ is the number of years since 1990. Write a polynomial $T(x)$ that can be used to model Mr. Silva's total manufacturing costs.**

Total cost is the product of the number of units and the cost per unit.

$T(x) = N(x) \cdot C(x)$.

Multiply the two polynomials.

$$\begin{array}{r} 0.02x^2 + 0.2x + 3 \\ \underline{\times\ -0.002x^2 - 0.1x + 2} \\ 0.04x^2 + 0.4x + 6 \\ -0.002x^3 - 0.02x^2 - 0.3x \phantom{+ 6} \\ \underline{-0.00004x^4 - 0.0004x^3 - 0.006x^2 \phantom{+ 0.4x + 6}} \\ -0.00004x^4 - 0.0024x^3 + 0.014x^2 + 0.1x + 6 \end{array}$$

Mr. Silva's total manufacturing costs, in thousands of dollars, can be modeled by $T(x) = -0.00004x^4 - 0.0024x^3 + 0.014x^2 + 0.1x + 6$.

CHECK IT OUT!

**3.** **What if...?** Suppose that in 2005 the cost of raw materials increases and the new average cost per unit is modeled by $C(x) = -0.004x^2 - 0.1x + 3$. Write a polynomial $T(x)$ that can be used to model the total costs.

You can also raise polynomials to powers.

## EXAMPLE 4 Expanding a Power of a Binomial

**Find the product.**

$(x + y)^3$

$(x + y)(x + y)(x + y)$ — *Write in expanded form.*

$(x + y)(x^2 + 2xy + y^2)$ — *Multiply the last two binomial factors.*

$x(x^2) + x(2xy) + x(y^2) + y(x^2) + y(2xy) + y(y^2)$ — *Distribute x and then y.*

$x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$ — *Multiply.*

$x^3 + 3x^2y + 3xy^2 + y^3$ — *Combine like terms.*

**CHECK IT OUT!** **Find each product.**

**4a.** $(x + 4)^4$ **4b.** $(2x - 1)^3$

Notice the coefficients of the variables in the final product of $(x + y)^3$. These coefficients are the numbers from the third row of Pascal's triangle.

| Binomial Expansion | Pascal's Triangle (Coefficients) |
|---|---|
| $(a + b)^0 = 1$ | 1 |
| $(a + b)^1 = a + b$ | 1 1 |
| $(a + b)^2 = a^2 + 2ab + b^2$ | 1 2 1 |
| $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ | 1 3 3 1 |
| $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$ | 1 4 6 4 1 |
| $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ | 1 5 10 10 5 1 |

Each row of Pascal's triangle gives the coefficients of the corresponding binomial expansion. The pattern in the table can be extended to apply to the expansion of any binomial of the form $(a + b)^n$, where $n$ is a whole number.

**Know it! Note**

### Binomial Expansion

For a binomial expansion of the form $(a + b)^n$, the following statements are true.

1. There are $n + 1$ terms.
2. The coefficients are the numbers from the $n$th row of Pascal's triangle.
3. The exponent of $a$ is $n$ in the first term, and the exponent decreases by 1 in each successive term.
4. The exponent of $b$ is 0 in the first term, and the exponent increases by 1 in each successive term.
5. The sum of the exponents in any term is $n$.

This information is formalized by the *Binomial Theorem*, which you will study further in Chapter 11.

## Student to Student

### *Expanding Binomials*

**Caitlin Humphrey**
Hillcrest High School

*I like to use a chart to expand binomials. I will use the binomial $(x + 2)^4$ as an example.*

*I write the coefficients from Pascal's triangle in the top row. I write the decreasing powers in the second row.*

*Then I shift one column to the right and write the increasing powers in the third row.*

| | | | | |
|---|---|---|---|---|
| 1 | 4 | 6 | 4 | 1 |
| $x^4$ | $x^3$ | $x^2$ | $x$ | |
| | 2 | 4 | 8 | 16 |

*Finally, I multiply vertically to get $x^4 + 8x^3 + 24x^2 + 32x + 16$.*

**EXAMPLE 5** **Using Pascal's Triangle to Expand Binomial Expressions**

**Expand each expression.**

**A** $(y - 3)^4$

**1 4 6 4 1** *Identify the coefficients for n = 4, or row 4.*

$$\left[1(y)^4(-3)^0\right] + \left[4(y)^3 - 3^1\right] + \left[6(y)^2(-3)^2\right] + \left[4(y)^1(-3)^3\right] + \left[1(y)^0(-3)^4\right]$$

$y^4 - 12y^3 + 54y^2 - 108y + 81$

**B** $(4z + 5)^3$

**1 3 3 1** *Identify the coefficients for n = 3, or row 3.*

$$\left[1(4z)^3(5)^0\right] + \left[3(4z)^2(5)^1\right] + \left[3(4z)^1(5)^2\right] + \left[1(4z)^0(5)^3\right]$$

$64z^3 + 240z^2 + 300z + 125$

**CHECK IT OUT!** **Expand each expression.**

**5a.** $(x + 2)^3$ **5b.** $(x - 4)^5$ **5c.** $(3x + 1)^4$

## THINK AND DISCUSS

1. The product of $(3x^4 - 2x^2 - 1)$ and a polynomial $P(x)$ results in a polynomial of degree 9. What is the degree of $P(x)$? Explain.
2. After $(2x + 8)^7$ is expanded, what is the degree of the result, and how many terms does the result have? Explain.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example and find the product.

Know it! Note