3-2 Using Algebraic Methods to Solve Linear Systems

TEKS 2A.3.B Foundations for functions: use algebraic methods, graphs, tables, or matrices to solve systems of equations or inequalities.

Objectives
Solve systems of equations by substitution.

Solve systems of equations by elimination.

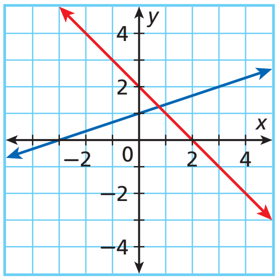
Vocabulary
substitution
elimination

Also 2A.1.A, 2A.3.A, 2A.3.C, 2A.4.A

Who uses this?

Zookeepers use algebraic methods to solve systems of linear equations that model mixtures of animal foods. (See Example 4.)

The graph shows a system of linear equations. As you can see, without the use of technology, determining the solution from the graph is not easy. You can use the *substitution* method to find an exact solution. In **substitution**, you solve one equation for one variable and then substitute this expression into the other equation.



EXAMPLE 1 Solving Linear Systems by Substitution

Use substitution to solve each system of equations.

A $\begin{cases} y = x + 2 \\ x + y = 8 \end{cases}$

Step 1 Solve one equation for one variable.

The first equation is already solved for y: $y = x + 2$.

Step 2 Substitute the expression into the other equation.

$x + y = 8$

$x + (x + 2) = 8$ *Substitute $(x + 2)$ for y in the other equation.*

$2x + 2 = 8$ *Combine like terms.*

$2x = 6$

$x = 3$

Step 3 Substitute the x-value into one of the original equations to solve for y.

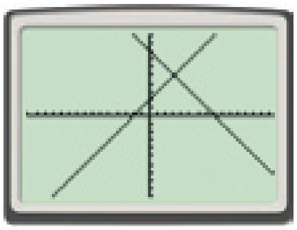
$y = x + 2$

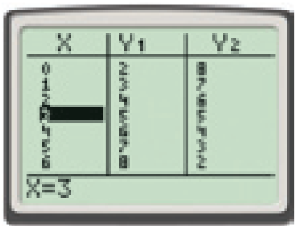
$y = (3) + 2$ *Substitute $x = 3$.*

$y = 5$

The solution is the ordered pair $(3, 5)$.

Check A graph or table supports your answer.





Caution!

The solution to an independent system of equations is an ordered pair. Do not stop working when you have found only one value.

Use substitution to solve each system of equations.

B $\begin{cases} 2x + y = 6 \\ y - 8x = 1 \end{cases}$

Method 1 Isolate y.

$2x + y = 6$ *First equation*

$y = 6 - 2x$ *Isolate one variable.*

$y - 8x = 1$ *Second equation*

$(6 - 2x) - 8x = 1$ *Substitute the expression into the second equation.*

$6 - 10x = 1$ *Combine like terms.*

$-10x = -5$

$x = \frac{1}{2}$ *First part of the solution*

Method 2 Isolate x.

$2x + y = 6$

$x = \frac{6 - y}{2} = 3 - \frac{y}{2}$

$y - 8x = 1$

$y - 8\left(3 - \frac{y}{2}\right) = 1$

$y - 24 + 4y = 1$

$5y = 25$

$y = 5$

Substitute the value into one of the original equations to solve for the other variable.

Method 1:

$y - 8\left(\frac{1}{2}\right) = 1$ *Substitute the value to solve for the other variable.*

$y - 4 = 1$

$y = 5$ *Second part of the solution*

Method 2:

$2x + (5) = 6$

$2x = 1$

$x = \frac{1}{2}$

By either method, the solution is $\left(\frac{1}{2}, 5\right)$.

CHECK IT OUT! **Use substitution to solve each system of equations.**

1a. $\begin{cases} y = 2x - 1 \\ 3x + 2y = 26 \end{cases}$

1b. $\begin{cases} 5x + 6y = -9 \\ 2x - 2 = -y \end{cases}$

You can also solve systems of equations with the *elimination* method. With **elimination**, you get rid of one of the variables by adding or subtracting equations. You may have to multiply one or both equations by a number to create variable terms that can be eliminated.

EXAMPLE 2 Solving Linear Systems by Elimination

Reading Math

The elimination method is sometimes called the *addition method* or *linear combination.*

Use elimination to solve each system of equations.

A $\begin{cases} 2x + 3y = 34 \\ 4x - 3y = -4 \end{cases}$

Step 1 Find the value of one variable.

$2x + 3y = 34$ *The y-terms have opposite coefficients.*

$+\ 4x - 3y = -4$

$6x \quad = 30$ *Add the equations to eliminate y.*

$x = 5$ *First part of the solution*

Step 2 Substitute the x-value into one of the original equations to solve for y.

$2(5) + 3y = 34$

$3y = 24$

$y = 8$ *Second part of the solution*

The solution to the system is $(5, 8)$.

Use elimination to solve each system of equations.

B $\begin{cases} 2x + 4y = -10 \\ 3x + 3y = -3 \end{cases}$

Step 1 To eliminate x, multiply both sides of the first equation by 3 and both sides of the second equation by -2.

$$\begin{aligned} 3(2x + 4y) &= 3(-10) \\ -2(3x + 3y) &= -2(-3) \end{aligned} \rightarrow \begin{aligned} 6x + 12y &= -30 \\ -6x - 6y &= 6 \\ \hline 6y &= -24 \end{aligned}$$ *Add the equations.*

$y = -4$ *First part of the solution*

Step 2 Substitute the y-value into one of the original equations to solve for x.

$3x + 3(-4) = -3$

$3x - 12 = -3$

$3x = 9$

$x = 3$ *Second part of the solution*

The solution to the system is $(3, -4)$.

Check Substitute 3 for x and -4 for y in each equation.

$2x + 4y = -10$	
$2(3) + 4(-4)$	-10
-10	-10 ✔

$3x + 3y = -3$	
$3(3) + 3(-4)$	-3
-3	-3 ✔

CHECK IT OUT!

Use elimination to solve each system of equations.

2a. $\begin{cases} 4x + 7y = -25 \\ -12x - 7y = 19 \end{cases}$ **2b.** $\begin{cases} 5x - 3y = 42 \\ 8x + 5y = 28 \end{cases}$

In Lesson 3-1, you learned that systems may have infinitely many or no solutions. When you try to solve these systems algebraically, the result will be an identity or a contradiction.

EXAMPLE 3 Classifying Systems with Infinitely Many or No Solutions

Remember!

An *identity*, such as $0 = 0$, is always true and indicates infinitely many solutions.
A *contradiction*, such as $1 = 3$, is never true and indicates no solution.

Classify the system and determine the number of solutions.

$\begin{cases} 2x + y = 8 \\ 6x + 3y = -15 \end{cases}$

Because isolating y is straightforward, use substitution.

$2x + y = 8$

$y = 8 - 2x$ *Solve the first equation for y.*

$6x + 3(8 - 2x) = -15$ *Substitute 8 − 2x for y in the second equation.*

$6x + 24 - 6x = -15$ *Distribute.*

$24 = -15$ ✘ *Simplify.*

Because 24 is never equal to -15, the equation is a contradiction. Therefore, the system is inconsistent and has no solution.

CHECK IT OUT!

Classify the system and determine the number of solutions.

3a. $\begin{cases} 56x + 8y = -32 \\ 7x + y = -4 \end{cases}$ **3b.** $\begin{cases} 6x + 3y = -12 \\ 2x + y = -6 \end{cases}$

EXAMPLE 4 *Zoology Application*

Texas Link: Zoology

The Fort Worth Zoo is home to a colony of prairie dogs in its High Plains & Prairies habitat. The prairie dogs share their space with Texas-native foxes, ferrets, and burrowing owls.

A zookeeper needs to mix feed for the prairie dogs so that the feed has the right amount of protein. Feed A has 12% protein. Feed B has 5% protein. How many pounds of each does he need to mix to get 100 lb of feed that is 8% protein?

Let a represent the amount of feed A in the mixture.

Let b represent the amount of feed B in the mixture.

Write one equation based on the amount of feed:

Amount of feed A	plus	amount of feed B	equals	100.
a	$+$	b	$=$	100

Write another equation based on the amount of protein:

Protein of feed A	plus	protein of feed B	equals	protein in mixture.
$0.12a$	$+$	$0.05b$	$=$	$0.08(100)$

Solve the system. $\begin{cases} a + b = 100 \\ 0.12a + 0.05b = 8 \end{cases}$

$a + b = 100$ *First equation*

$b = 100 - a$ *Solve the first equation for b.*

$0.12a + 0.05(100 - a) = 8$ *Substitute (100 − a) for b.*

$0.12a + 5 - 0.05a = 8$ *Distribute.*

$0.07a = 3$ *Simplify.*

$a \approx 42.9$ *Round to the nearest tenth.*

Substitute a into one of the original equations to solve for b.

$(42.9) + b \approx 100$ *Substitute the value of a into one equation.*

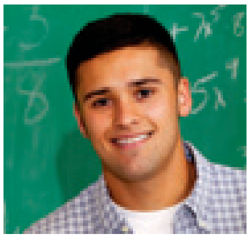
$b \approx 57.1$ *Solve for b.*

The mixture will contain about 42.9 lb of feed A and 57.1 lb of feed B.

CHECK IT OUT!

4. A coffee blend contains Sumatra beans, which cost \$5/lb, and Kona beans, which cost \$13/lb. If the blend costs \$10/lb, how much of each type of coffee is in 50 lb of the blend?

Student to Student — *Solving Systems*



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Choosing a method to solve a system of linear equations can be confusing. Here is how I decide which method to use:

Graphing and tables*— when I'm interested in a rough solution or other values around the solution*

Substitution*— when it's simple to solve one of the equations for one variable (for example, solving $3x + y = 7$ for y)*

Elimination*— when variables have opposite coefficients, like 5x and −5x, or when I can easily multiply the equations to get opposite coefficients*