3-1 Using Graphs and Tables to Solve Linear Systems

TEKS 2A.3.B Foundations for functions: use algebraic methods, graphs, tables, or matrices to solve systems of equations or inequalities.

Objectives

Solve systems of equations by using graphs and tables.

Classify systems of equations, and determine the number of solutions.

Vocabulary

system of equations
linear system
consistent system
inconsistent system
independent system
dependent system

Also 2A.1.A, 2A.3.A, 2A.3.C, 2A.4.A

Who uses this?

Winter sports enthusiasts can use systems of equations to compare the costs of renting snowboards. (See Example 4.)

A **system of equations** is a set of two or more equations containing two or more variables. A **linear system** is a system of equations containing only linear equations.

Recall that a line is an infinite set of points that are solutions to a linear equation. The solution of a system of equations is the set of all points that satisfy each equation.

On the graph of the system of two equations, the solution is the set of points where the lines intersect. A point is a solution to a system of equations if the x- and y-values of the point satisfy both equations.

EXAMPLE 1 Verifying Solutions of Linear Systems

Use substitution to determine if the given ordered pair is an element of the solution set for the system of equations.

A $(2, 4); \begin{cases} x - 2y = -6 \\ 2x + y = 8 \end{cases}$

$$\begin{array}{r|l} \multicolumn{2}{c}{x - 2y = -6} \\ \hline (2) - 2(4) & -6 \\ -6 & -6 \ \checkmark \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{2x + y = 8} \\ \hline 2(2) + (4) & 8 \\ 8 & 8 \ \checkmark \end{array}$$

Substitute 2 for x and 4 for y in each equation.

Because the point is a solution of both equations, it is a solution of the system.

B $(3, 2); \begin{cases} 2x + 3y = 12 \\ 8x - 6y = 24 \end{cases}$

$$\begin{array}{r|l} \multicolumn{2}{c}{2x + 3y = 12} \\ \hline 2(3) + 3(2) & 12 \\ 12 & 12 \ \checkmark \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{8x - 6y = 24} \\ \hline 8(3) + 6(2) & 24 \\ 36 & 24 \ \times \end{array}$$

Substitute 3 for x and 2 for y in each equation.

Because the point is not a solution of both equations, it is not a solution of the system.

CHECK IT OUT! **Use substitution to determine if the given ordered pair is an element of the solution set for the system of equations.**

1a. $(4, 3); \begin{cases} x + 2y = 10 \\ 3x - y = 9 \end{cases}$

1b. $(5, 3); \begin{cases} 6x - 7y = 1 \\ 3x + 7y = 5 \end{cases}$

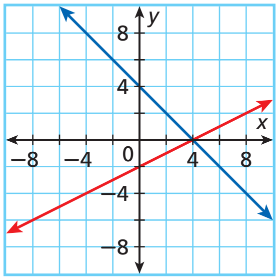
Recall that you can use graphs or tables to find some of the solutions to a linear equation. You can do the same to find solutions to linear systems.

EXAMPLE 2 Solving Linear Systems by Using Graphs and Tables

Use a graph and a table to solve each system. Check your answer.

A $\begin{cases} x + y = 4 \\ 2y + 4 = x \end{cases}$

Solve each equation for y. $\begin{cases} y = -x + 4 \\ y = \frac{1}{2}x - 2 \end{cases}$



On the graph, the lines appear to intersect at the ordered pair $(4, 0)$.

Make a table of values for each equation. Notice that when $x = 4$, the y-value for both equations is 0.

$y = -x + 4$

x	y
1	3
2	2
3	1
4	**0**

$y = \frac{1}{2}x - 2$

x	y
1	$-\frac{3}{2}$
2	-1
3	$-\frac{1}{2}$
4	**0**

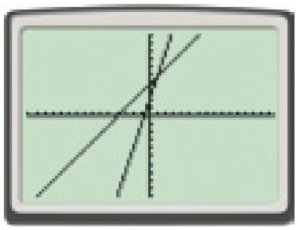
The solution to the system is $(4, 0)$.

B $\begin{cases} 3x - y = -2 \\ x - y = -4 \end{cases}$

Solve each equation for y. $\begin{cases} y = 3x + 2 \\ y = x + 4 \end{cases}$

Use your graphing calculator to graph the equations and make a table of values. The lines appear to intersect at $(1, 5)$. This is confirmed by the table of values.

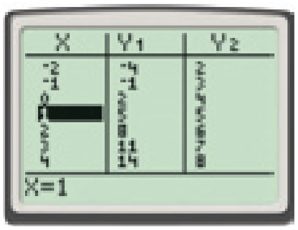
The solution to the system is $(1, 5)$.



Helpful Hint

To enter the equations into your calculator, let Y_1 represent $y = 3x + 2$ and let Y_2 represent $y = x + 4$.

Check Substitute $(1, 5)$ in the original equations to verify the solution.



$$\begin{array}{r|l} 3x - y & = -2 \\ \hline 3(1) - (5) & -2 \\ -2 & -2 \checkmark \end{array} \qquad \begin{array}{r|l} x - y & = -4 \\ \hline (1) - (5) & -4 \\ -4 & -4 \checkmark \end{array}$$

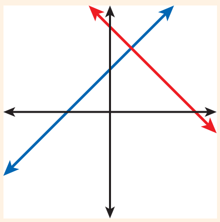
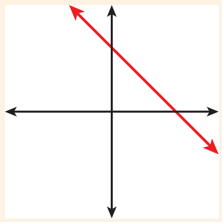
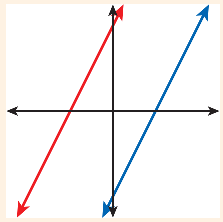
CHECK IT OUT! **Use a graph and a table to solve each system. Check your answer.**

2a. $\begin{cases} 2y + 6 = x \\ 4x = 3 + y \end{cases}$ **2b.** $\begin{cases} x + y = 8 \\ 2x - y = 4 \end{cases}$ **2c.** $\begin{cases} y - x = 5 \\ 3x + y = 1 \end{cases}$

The systems of equations in Example 2 have exactly one solution. However, linear systems may also have infinitely many or no solutions. A **consistent system** is a set of equations or inequalities that has at least one solution, and an **inconsistent system** will have no solutions.

You can classify linear systems by comparing the slopes and y-intercepts of the equations. An **independent system** has equations with different slopes. A **dependent system** has equations with equal slopes and equal y-intercepts.

Classifying Linear Systems

EXACTLY ONE SOLUTION	INFINITELY MANY SOLUTIONS	NO SOLUTION
		
Consistent, independent The graphs are intersecting lines with different slopes.	Consistent, dependent The graphs are coinciding lines; they have the same slope and same y-intercept.	Inconsistent The graphs are parallel lines; they have the same slope but different y-intercepts.

EXAMPLE 3 **Classifying Linear Systems**

Classify each system and determine the number of solutions.

A $\begin{cases} 2x + y = 3 \\ 6x = 9 - 3y \end{cases}$

Solve each equation for y. $\begin{cases} y = -2x + 3 \\ y = -2x + 3 \end{cases}$

The equations have the same slope and y-intercept and are graphed as the same line.

The system is dependent with infinitely many solutions.

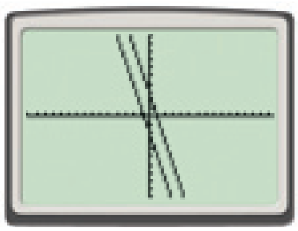
B $\begin{cases} 3x + y = 3 \\ 2 + y = -3x \end{cases}$

Solve each equation for y. $\begin{cases} y = -3x + 3 \\ y = -3x - 2 \end{cases}$

The equations have the same slope but different y-intercepts and are graphed as parallel lines.

The system is inconsistent and has no solution.

Check A graph shows parallel lines.



Remember!

The slope-intercept form of a linear equation makes comparing slopes and y-intercepts easy.

$y = mx + b$

Slope (m), y-intercept (b)

CHECK IT OUT! **Classify each system and determine the number of solutions.**

3a. $\begin{cases} 7x - y = -11 \\ 3y = 21x + 33 \end{cases}$ **3b.** $\begin{cases} x + 4 = y \\ 5y = 5x + 35 \end{cases}$

EXAMPLE 4 *Winter Sports Application*

Big Dog Snowboard Co. charges $15 for equipment rental plus $35 per hour for snowboarding lessons. Half-Pipe Snowboards, Inc. charges $40 for equipment rental plus $25 per hour for lessons. For what number of hours is the cost of equipment and lessons the same for each company?

Step 1 Write an equation for the cost of equipment rental and lessons at each company.

Let x represent the number of hours and y represent the total cost in dollars.

Big Dog Snowboard Co.: $y = 35x + 15$

Half-Pipe Snowboards, Inc.: $y = 25x + 40$

Because the slopes are different, the system is independent and has exactly one solution.

Step 2 Solve the system by using a table of values.

Use increments of $\frac{1}{2}$ to represent 30 min.

When $x = 2\frac{1}{2}$, the y-values are both 102.5. The cost of equipment rental and a $2\frac{1}{2}$-hour snowboard lesson is $102.5 at either company. So the cost is the same at each company for $2\frac{1}{2}$ hours.

$y = 35x + 15$

x	y
1	50
$1\frac{1}{2}$	67.5
2	85
$2\frac{1}{2}$	102.5
3	120

$y = 25x + 40$

x	y
1	65
$1\frac{1}{2}$	77.5
2	90
$2\frac{1}{2}$	102.5
3	115

CHECK IT OUT!

4. Ravi is comparing the costs of long distance calling cards. To use card A, it costs $0.50 to connect and then $0.05 per minute. To use card B, it costs $0.20 to connect and then $0.08 per minute. For what number of minutes does it cost the same amount to use each card for a single call?

THINK AND DISCUSS

1. Explain how to find the number of solutions of a system of equations using only a graph.

2. Explain why a system of equations whose graphs are distinct parallel lines has no solution.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, give information about or examples of each solution type.

	Exactly One Solution	Infinitely Many Solutions	No Solution
Example			
Graph			
Slopes			
y-intercepts			